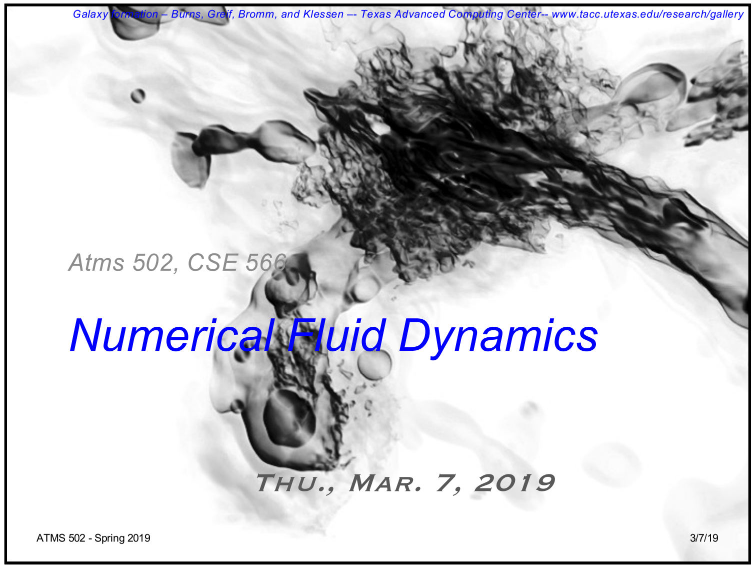

Atms 502, CSE 566

Numerical Fluid Dynamics

THU., MAR. 7, 2019

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ATMS 502
CSE 566

Thursday,
7 March 2019

Class #16

Plan for Today

- 1) Skamarock & Klemp
  - Global/local refinement
  - A full AMR method, explained
- 1) Space differencing
  - Differential-difference approach
  - Phase speed & group velocity
  - Impacts of higher order, added diffusion
- 2) Programming input/output
  - C and Fortran considerations
- 3) Program 4: *continued*
  - Nest placement / movement

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Review from last class

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Review: Space Differencing

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- Differential-difference eqn:
  - 2nd-order space: $\frac{du_j}{dt} + c\left(\frac{u_{j+1} - u_{j-1}}{2\Delta x}\right) = 0$ Substitute: $u_j(t) = e^{i(kj\Delta x - \omega_{2c} t)}$
  - Solve for frequency ω, which *may be complex;* here $\omega = \frac{c\sin\beta}{\Delta x}$
- Once we have the frequency:
  - Phase speed: $c_{2c} = \frac{\omega}{k} = \frac{c\sin\beta}{k\Delta x}$; for small kΔx, $c_{2c} \approx c\left(1 - \frac{\beta^2}{6}\right)$
    - perfect (=c) for infinite waves where kΔx=0
    - zero for 2Δx waves
  - Group velocity: $c_{g2c} = \frac{\partial\omega}{\partial k} = \frac{\partial}{\partial k}\left(\frac{c\sin\beta}{\Delta x}\right) = c(\cos\beta)$
    - perfect for infinite waves, = -c for 2Δx waves.

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Review: 2nd vs. 4th order space diff.

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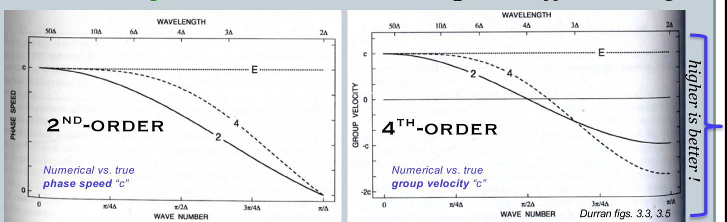
- *Summary: 2nd-order centered space differencing*



- Phase speed (left), group velocity (right)
- 4th better at "intermediate" wavelengths

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Review: Accuracy and Order

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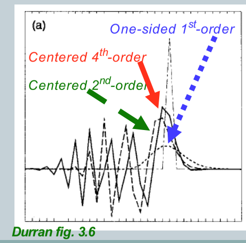
- "**Centered** [even-ordered] **schemes** preserve the amplitude of each individual Fourier component, [but] the various *components propagate at different speeds [and the result is ugly]*
- "Switching to a higher-order scheme does **not** improve the performance of finite-difference methods *when they are used to model poorly resolved features*



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Review: Explicit Artificial Dissipation

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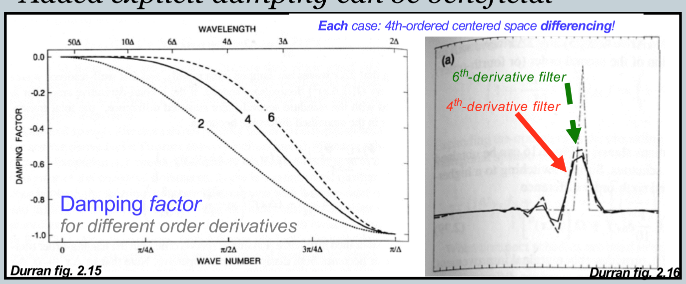
- *Added explicit damping can be beneficial*



- Higher order *derivatives [in damping]* damp intermediate wavelengths *less.*
- Added damping can reduce the dispersion errors.

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Skamarock & Klemp

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ADAPTIVE MESH REFINEMENT PAPER

SEE NOTES FROM LAST CLASS!

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Program 4

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NEST PLACEMENT ALGORITHM

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Nest location

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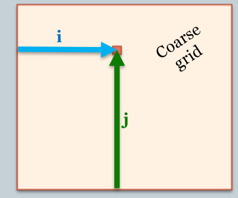
- About that nest location -
  - This is what my code sequence looks like, in the routine in which I compute the truncation error.




```
loop: all i = 3:nx-2
  loop: all j = 3:ny-2
    xtrunc = (expression in X direction: i-2, i-1, i, i+1, i+2)
    ytrunc = (expression in Y direction: j-2, j-1, j, j+1, j+2)
    trunc_error(i,j) = max( abs(xtrunc) , abs(ytrunc) )
  end loop j
end loop i
```

  - Now I work with trunc_error() array - get max value, find truncation error "edges", average to find nest **center**.

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Determining the nest location (2)

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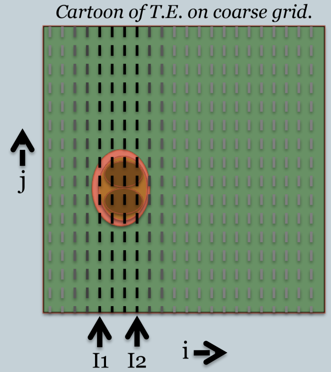
Cartoon of T.E. on coarse grid.



- Example of finding left, right edges of T.E. region
  - For each *i* column, left to right … check TE(i,j) for all j rows – determine max value
  - Find first and last column (i) for which max ≥ threshold.
- Do same for top/bottom.
- Average I1,I2,J1,J2 for nest *center*. Nest *edges* depend on nest *size*.

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Program 4 questions (1)

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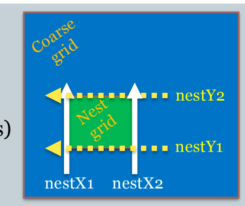
- Handling the nest
  - four variables define *edges* of the nest
    - these variables are integers holding the *coarse grid coordinate* of the nest (discuss)
  - when *placing the nest* for the first time, *setting boundary values* for the nest, *doing feedback* …
    - then these four are the variables *dointerp* uses.
    - the other variables *nestX1old, nestX2old, nestY1old*… are ignored.
  - when *moving* the nest:
    - nestX1, nestX2, nestY1, nestY2 = location *where nest is to be moved*
    - nestX1old, nestX2old, etc … contain *old (current) nest location*



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Program 4 Steps, part 1

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1. You need rotational flow I.C. code (from program 2)
2. Copy my program 4 files on Stampede
3. Try interpolation code (F90 or C)
4. "Run" code with a passive "nest"

   } *Simply use cone center for nest location; then interpolate coarse > nest*
5. Develop truncation error code
   (find x, y truncation error, store 2d array of max(abs(xerror,yerror))...
6. Determine nest location
   get x,y error bounds; average = nest center; determine X1,X2,Y1,Y2

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Program 4 Steps, part 2

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7. Alter code to *place* nest at start (n=1);
   move nest afterwards (at time step n=5,10,...)
8. Boundary conditions for nest
   (Easy: Call dointerp with flag to just set nest BCs)
9. Alter integrate routine for nest
   (This is just a matter of what points are updated – 2:nx-1 on nest, etc)
10. Evolve the nest
    No feedback! Just get B.C's from coarse grid, integrate/update nest
11. Add feedback.
    By now you know your nest is 'ok' – do this step last.

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Program 4 variables

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- **You need to add:**
  - Extra storage (s1, s2 for nested grid)
  - Nest-specific parameters
    - Time step for nest
    - Grid spacing for nest
  - New variables – ***read these in***
    - Grid refinement ratio *(an integer. time & space!)*
    - Current nest location info *(first, last grid points in X,Y)*
    - Nest *update frequency (an integer; 1 = every step)*
    - Feedback *option*
      - *on or off.*

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Program 4 Code restructuring

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- **Changes to problem 3 layout:**
  - Just *one* advection method *(Lax-W)*
  - Advection routine called for both grids
    - Boundary condition will differ between grids
  - Additional time loop for nested grid
    - Boundary conditions *from coarse grid*
    - Many *nested grid steps* for each coarse grid step
  - Feedback code
  - Error calculations

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Finite volume method; van Leer

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Overview:

- van Leer published five papers between 1973-79
- J. Comp. Phys., vol. 23, 276-299 (1977):

  "Towards the ultimate conservative difference scheme: IV. A new approach to numerical convection"

Handout:

- Hourdin and Armengaud, 1999: Use of finite volume methods for atmospheric advection of trace species
  - flux forms
  - monotonicity
  - "approximating the sub-grid-scale distribution by a polynomial" *(HA99 p. 823)*

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van Leer (1977)

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- This all applies to the flux form of our equations e.g.

$$\frac{\partial q}{\partial t} = -u\frac{\partial q}{\partial x} - w\frac{\partial q}{\partial z} \Rightarrow$$

$$\frac{\partial q}{\partial t} = -\frac{\partial}{\partial x}(uq) - \frac{\partial}{\partial z}(wq) + q\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)$$

- Conceptually:
  - grid point values are averages within a grid zone
  - local functions describe field changes within the zone
    - piecewise *constant*, piecewise *linear*, piecewise *parabolic*
  - we integrate under local functions at time t *that will be* in grid zone of interest [0,Δx] *at t+Δt.*

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Upstream – piecewise *constant*

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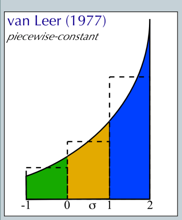
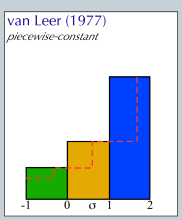
- Step 1 –
- identify grid zone averages...
- X coordinate is Courant number σ
- Grid box runs from [0-to-1]•Δx

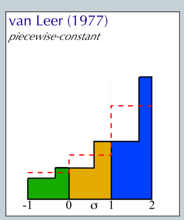
- Step 2 –
- look at distribution **before** and after advection takes place

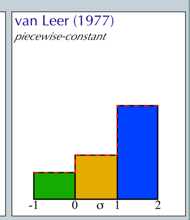
- Step 3 -
- Compute new grid zone averages.

- Step 4 -
- The averages *are* the function here (for piecewise constant)
- These are the initial values for the next time step.









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van Leer (1977)

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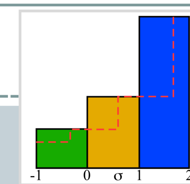

- New value from integrating under piecewise constant function at time *t* that *will be in the grid zone [0,Δx] at t+Δt.*

$$q^{n+1} \equiv \bar{q}^{1/2} = \int_0^{1-\sigma} q_{1/2}\,dx + \int_{-\sigma}^{0} q_{-1/2}\,dx \qquad \sigma = \frac{u\Delta t}{\Delta x}$$

Grid-point value f(j) represents the average of the function over the grid cell (see Durran, § 1.3.1, p. 27)

- Piecewise *constant* in each zone, so:

$$\bar{q}^{1/2} = \bar{q}_{1/2}(1-\sigma) + \bar{q}_{-1/2}\sigma = \bar{q}_{1/2} - \sigma(\bar{q}_{1/2} - \bar{q}_{-1/2}) \quad \longleftrightarrow \quad \bar{q}^{1/2} = \bar{q}_{1/2} - \left[u\bar{q}_{1/2} - u\bar{q}_{-1/2}\right]\frac{\Delta t}{\Delta x}$$

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van Leer (1977)

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- van Leer notation ...

$$\bar{q}^{1/2} = \bar{q}_{1/2} - \sigma(\bar{q}_{1/2} - \bar{q}_{-1/2}) = \bar{q}_{1/2} - [\sigma\bar{q}_{1/2} - \sigma\bar{q}_{-1/2}] = \bar{q}_{1/2} - [Flux_{1/2} - Flux_{-1/2}]$$

- Fluxes ...

$$Flux(i) \equiv Flux_{-1/2} = \sigma q_{-1/2} = \sigma q(i-1)$$

  - note Δt is already included in (is part of) the fluxes.

- Coding:

$$q^{n+1} = q^n - (Flux_{1/2} - Flux_{-1/2}) + q_i^n \Delta t \delta_x u$$
$$= q^n - [(\sigma q_i) - (\sigma q_{i-1})] + q_i^n \frac{\Delta t}{\Delta x}(u_{i+1} - u_i)$$
$$= q^n - \sigma(q_i - q_{i-1}) \quad if \;\; u = \text{constant}$$

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Greater accuracy: *Piecewise linear*

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- Our general update formula:

$$q^{n+1} \equiv \bar{q}^{1/2} = \int_0^{1-\sigma} q_{1/2}dx + \int_{-\sigma}^{0} q_{-1/2}dx \quad \sigma = \frac{u\Delta t}{\Delta x}$$

- Piecewise *constant*
  $q(x,t_0) = \bar{q}_{1/2}$
- Piecewise *linear*
  $q(x,t_0) = \bar{q}_{1/2} + \bar{\Delta}_{1/2}q\left(x - \frac{1}{2}\right)$
- Includes:
  - Zone average $\bar{q}_{1/2} = \int_0^1 q(x,t^0)dx$
  - Average slope $\bar{\Delta}_{1/2}q \equiv \left(\partial q/\partial x\right)_{1/2}$ ← Constant – why?

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Piecewise linear

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- Van Leer piecewise linear: *Scheme I*

$$\bar{q}^{1/2} = \bar{q}_{1/2} - \sigma(\bar{q}_{1/2} - \bar{q}_{-1/2}) - \frac{\sigma}{2}(1-\sigma)(\bar{\Delta}_{1/2}q - \bar{\Delta}_{-1/2}q)$$

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Piecewise linear & beyond

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- Van Leer piecewise linear: *Scheme I*

$$\bar{q}^{1/2} = \bar{q}_{1/2} - \sigma(\bar{q}_{1/2} - \bar{q}_{-1/2}) - \frac{\sigma}{2}(1-\sigma)(\bar{\Delta}_{1/2}q - \bar{\Delta}_{-1/2}q)$$

Like piecewise-constant: fluxes from zone averages.

$$\bar{q}_{1/2} = \int_0^1 q(x,t^0)dx$$

Slopes in zones. - can be evaluated many ways. Scheme 1: centered differences

$$\bar{\Delta}_{1/2}q = \frac{1}{2}(\bar{q}_{3/2} - \bar{q}_{-1/2})$$

- How could we improve our method?
  - 1. ____________________
  - 2. ____________________

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