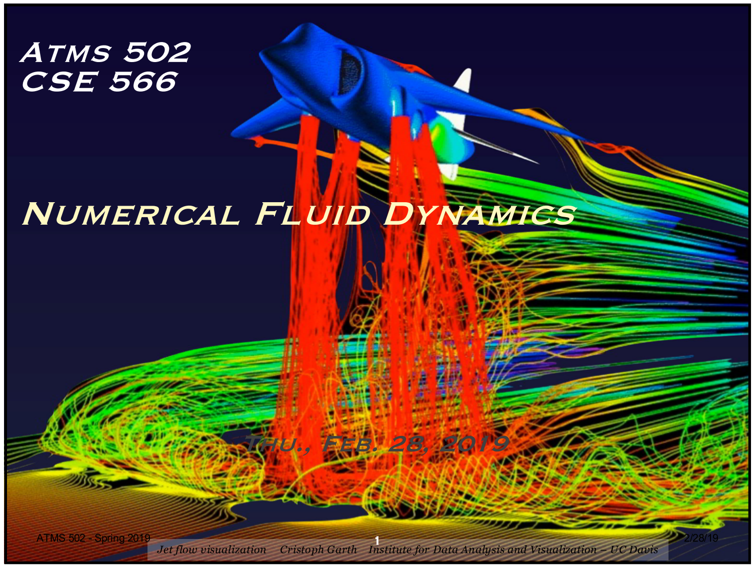

Plan for Today

ATMS 502
CSE 566

Thursday,
28 February 2019

Class #14

- Pgm3 due Wed Mar. 6

- 1) Time filtering
  - Damping leapfrog's computational mode
- 2) Grid refinement & clustering
  - Skamarock dissertation (notes: last class)
- 3) Program 4: *provided codes*
  - Placing/moving nest, & feedback
- 4) Resolution
  - Resolved/*permitted;* KE spectra method
- 5) Nesting, continued
  - Some questions & answers

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Leapfrog stability - review

- We rewrote the 3-level scheme as 2-level:

$$\left.\begin{array}{l}\tilde{u}^{n+1}=\tilde{v}^{n}-\mu\tilde{u}^{n}(2i\sin\beta)\\ \tilde{v}^{n+1}=\tilde{u}^{n}\end{array}\right\} \text{ so } \begin{pmatrix}\tilde{u}^{n+1}\\ \tilde{v}^{n+1}\end{pmatrix}=\begin{pmatrix}-2i\mu\sin\beta & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\tilde{u}^{n}\\ \tilde{v}^{n}\end{pmatrix}$$

- Write above as matrix, subtract l from diagonal, set determinant to zero. Characteristic equation:

$$\begin{vmatrix}-2i\mu\sin\beta-\lambda & 1\\ 1 & 0-\lambda\end{vmatrix}=0$$

- Solve; 2 roots; physical and computational modes

$$\lambda=-i\mu\sin\beta\pm\sqrt{1-\mu^2\sin^2\beta}=-ip\pm\sqrt{1-p^2}$$

- As Δt and p ⇒0: "+" root approaches 1, "-" root: -1
  - $|\lambda|=-1$ means amplitude varies as $(-1)^n$

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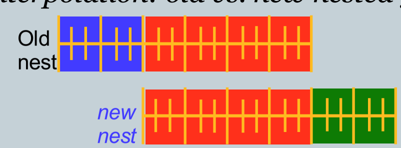
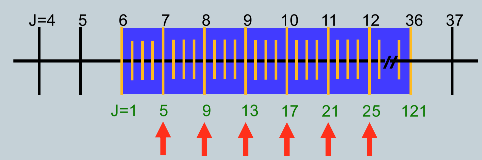
Review: 1-D Nesting

- *Interpolation: old vs. new nested grids*



THIS EXAMPLE: 3:1 NESTING

- *Feedback: copy nested ⇒ coarse*



THIS EXAMPLE: 4:1 NESTING

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Time filtering

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LEAPFROG TIME+SPACE DIFFERENCING
PHYSICAL AND COMPUTATIONAL MODES
UNDAMPED COMPUTATIONAL MODES
TIME FILTERING: *WHY*, AND *HOW TO?*

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Time-filtered Leapfrog

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- Advantages of the Leapfrog method:
  - Stable, 2nd order
  - Simple, & thus computationally cheap
    - but little computation for amount of communication
    - this is true for other schemes we have examined, too.
  - *No* amplitude error (if stable)
- Disadvantages:
  - Undamped computational mode
    - How to find the physical vs. computational mode
    - What is an *undamped* computational mode?
    - Odd/even solutions; may diverge
  - Dispersion, etc *(not unique to leapfrog)*

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Time-filtered Leapfrog

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- How to control the computational mode?
  - Periodically discard (n-1) time level data
  - Restart integration with a 2-level scheme
    - Common practice: **FTCS** scheme *(forward time, centered space)*
    - ... but **FTCS** is unstable, and
    - ... **FTCS** is 1st order (degrades accuracy)
    - Or: use Upstream or Lax-Wendroff
- Time smoothing
  - Remember computational mode: $\lambda \sim (-1)^n$
  - Smooth across (n-1, n, n+1) time levels

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Time-filtered Leapfrog

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- Time smoother for Leapfrog *(Asselin 1972)*
  - Instead of:

$$u_j^{n+1} = u_j^{n-1} - \mu\left(u_{j+1}^n - u_{j-1}^n\right)$$

  - Time smoothing:

$$u_j^{n+1} = \overline{u_j^{n-1}} - \mu\left(u_{j+1}^n - u_{j-1}^n\right) \quad \text{(LEAPFROG STEP)}$$

$$\overline{u_j^n} = u_j^n + \varepsilon\left(u_j^{n+1} - 2u_j^n + \overline{u_j^{n-1}}\right) \quad \text{(SMOOTHING STEP)}$$

  - Stable if $\mu<(1-\varepsilon)$
    - So there is a more restrictive stability condition.

ATMS 502 - Spring 2019 C055: Computational modes 2/28/19

Time-filtered Leapfrog

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- Time smoother for Leapfrog *(Asselin 1972)*

$$u_j^{n+1} = \overline{u_j^{n-1}} - \mu\left(u_{j+1}^n - u_{j-1}^n\right) \quad \text{(Leapfrog step)}$$

$$\overline{u_j^n} = u_j^n + \varepsilon\left(u_j^{n+1} - 2u_j^n + \overline{u_j^{n-1}}\right) \quad \text{(Smoothing step)}$$

- Sequence:
  - Have: (n-1, smoothed) and (n, unsmoothed)
  - Take leapfrog step to get (n+1, unsmoothed)
  - Use new (n+1, unsmoothed) to smooth u(n)
  - Ready for next step [smoothed u => u(n-1)]

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Grid refinement & Clustering

Notes – handed out last class!

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ADAPTIVE MESH REFINEMENT

Reference pages for this section:

- C008 – Truncation error
- C009 – Resolution
- C010 – AMR / nesting
- C051 – Nesting: grid placement, movement

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Program 4

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NESTING TOOLS PROVIDED TO YOU

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Program #4: main routine

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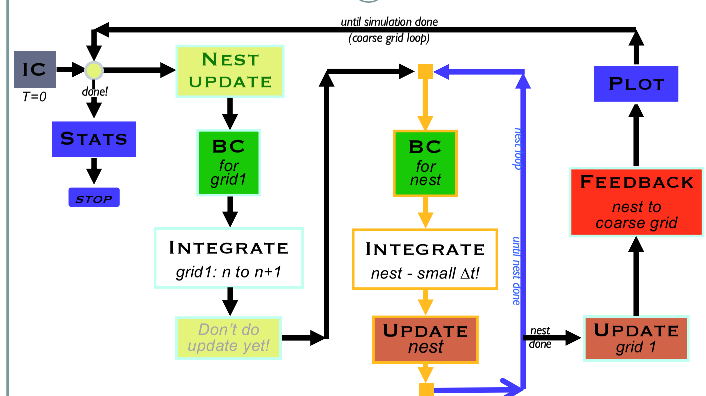

ATMS 502 - Spring 2019 C049: Nesting – coding and sequence of operations 2/28/19

Nest BCs

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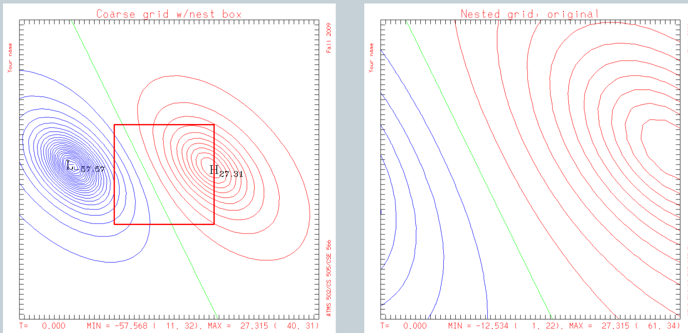
- “Placing” the nest

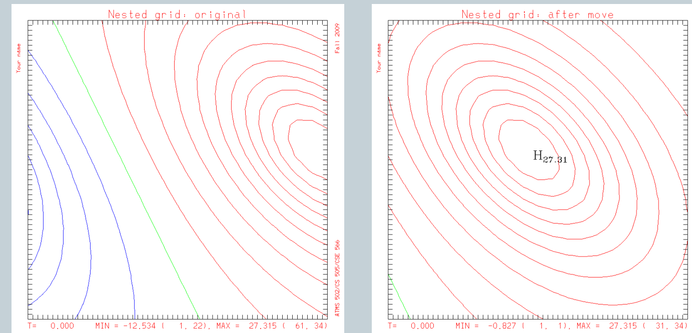


ATMS 502 - Spring 2019 Coarse grid Nested grid 2/28/19

Nest BCs

14

- Moving the nest

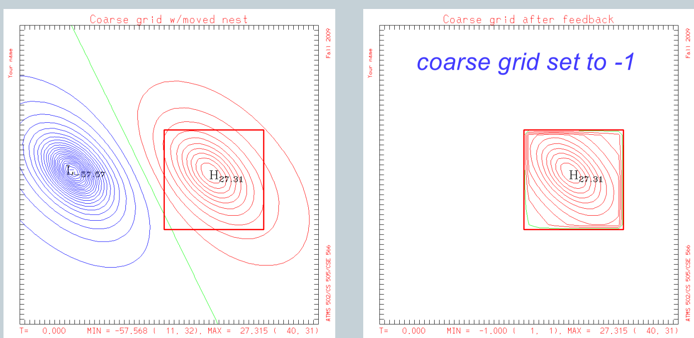


ATMS 502 - Spring 2019 Nest Nest moved 2/28/19

Nest BCs

15

- Testing feedback



ATMS 502 - Spring 2019 Coarse w/moved nest Test feedback to zero’d coarse 2/28/19

Resolution

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ATMS 502 - Spring 2019 C.009: Resolution 2/28/19

Resolving vs. *permitting*

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- Feature-*resolving* means what it says.
  - A model may instead have feature-*permitting* resolution
    - Means: those phenomena are "in" the model, e.g. via parameterizations and only in a very broad sense
  - Some models contain both *explicit* and *parameterized* physics
    - Explicit actually describes ~correct behavior – if really resolved
    - Parameterized reproduces bulk properties of the phenomena even though it is not resolved
    - Things get interesting in the *in-between* resolutions *("gray scales")*
- Liu and Moncrieff *(2007 Mon. Wea. Rev., p. 2866)*
  - Cloud-*permitting* runs "underperform" and exhibit greater sensitivity to parameterizations than the cloud-*resolving* models exhibit from their explicit physics.

ATMS 502 - Spring 2019 C.009: Resolution 2/28/19

Effective resolution

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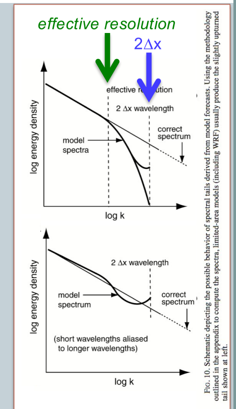
- Skamarock (2004):
  - Why move to higher resolution?
    - Typically this is done to resolve phenomena that are now marginally resolved, or unresolved (i.e. parameterized)
  - "Effective" resolution
    - There are known *kinetic energy spectra* profiles (see Skamarock Fig. 10 at right).
    - Models fail to reproduce these spectra at smaller scales. Note the dropoff at higher wavenumber (lower wavelengths)
    - He defines effective model resolution to be where the model spectra "decays"
      - WRF atmospheric model: $7\Delta x$ *(p. 3027)*



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Nesting

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ATMS 502 - Spring 2019 2/28/19

Question 1: oscillations

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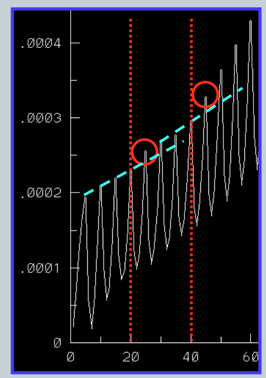
- Nest shock is superimposed on the high frequency oscillations seen here
- But: What causes oscillations?



- Total grid-1 error is plotted at each time step

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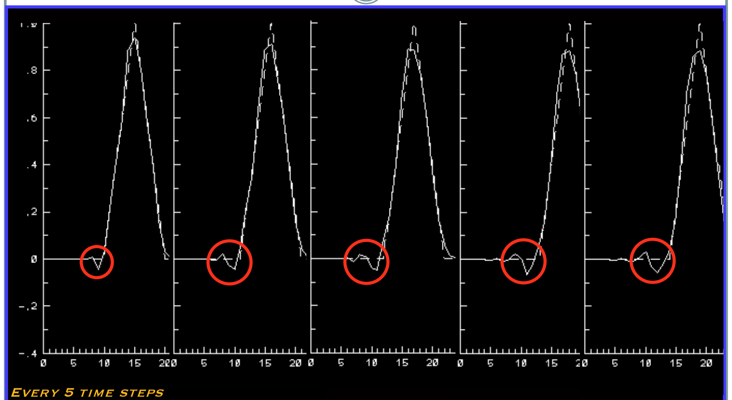
No, oscillations aren't due to *this*.

21



ATMS 502 - Spring 2019 C023: Phase error 2/28/19

Question 1: oscillations

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- How long does it take for features to move 1 Δx ???
  - *s=vt; time = distance/speed;* t=s/v
  - Total time to go distance Δx = $\Delta x/c$
  - # time steps to go Δx = 1/ν *(why?)*
- So, every 5 time steps … *(for ν=0.2)*
  - The cone peak moves one grid distance.
  - What does this say about our "true" solution?

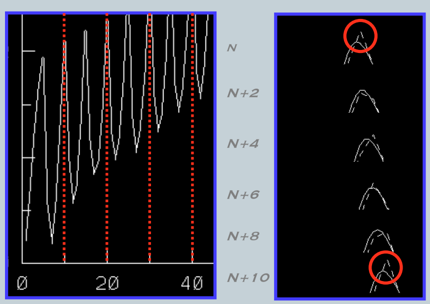
ATMS 502 - Spring 2019 C043: Courant number 2/28/19

Question: oscillations

23

(1) What *are* these oscillations?

- *Every 5 time steps the (exact) solution moves one grid length.*
- *At right: 2 periods in 10 steps*



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Question: oscillations

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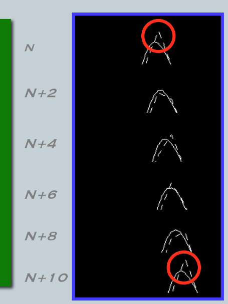
(1) What *are* these oscillations?

- *Every 5 time steps the (exact) solution moves one grid length.*
- *At right: 2 periods in 10 steps*

- *Projecting an exact solution on a finite grid results in errors - in the "true" solution !!*



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Question 2: update frequency

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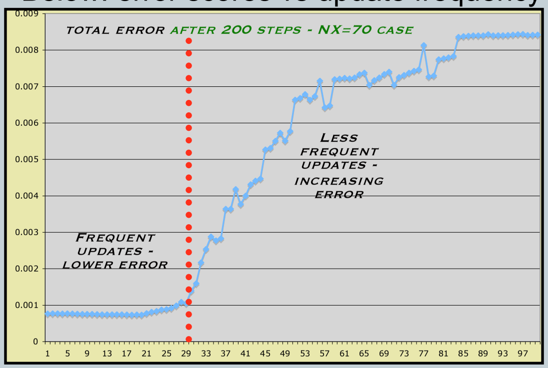
Below: error scores vs update frequency

Program 2 -- cone -- Lax-Wendroff



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Question 2: update frequency

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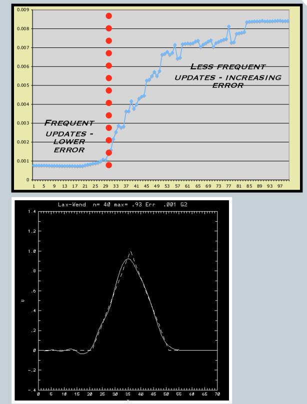

- Red dotted line is at ~30 (grid relocated every 30 steps).
- # steps for cone to move from center of nest to “center” of edge: *5*(24/2)=60*
- *What does this tell us?*

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Question 2: update frequency

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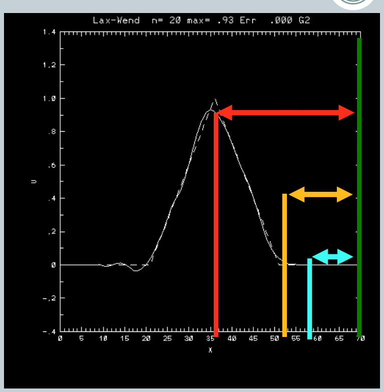

- In *60 steps* the middle of the cone is on the right nest domain edge.
- In *30 steps*, when the error starts rapidly rising, the *right edge* of the cone (and of trunc. error) hits the nest edge.
- In *20 steps*, the nest border is still ahead of the cone’s leading edge.

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The CFL condition

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STABILITY - CONTINUED

ATMS 502 - Spring 2019 2/28/19

PDE classification: Two kinds

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$$Au_{xx} + Bu_{xy} + Cu_{yy} = F(x,y,u,u_x,u_y)$$

Details: *F* is a *linear function* of u, ux, uy; coefficients may only depend on x, y. Subscripts are derivatives!

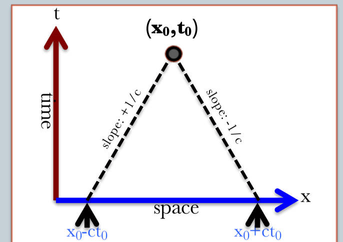
- Is *elliptic* if B^2-4AC < 0 — $u_{xx} + u_{yy} = 0$ — Laplace's equation
- Is *parabolic* if B^2-4AC = 0 — $u_t = c^2 u_{xx}$ — Heat equation
- Is *hyperbolic* if B^2-4AC > 0 — $u_{tt} = c^2 u_{xx}$ — Wave equation
- B^2-4AC came from *characteristics*, or characteristic curves - curves of *information propagation.* The 2nd-order wave equation has the characteristics: $x \pm ct$ = constant
- Initial value problems - the principal *computational* concern is the *stability* of the numerical algorithm/scheme.
  - PDE includes a time derivative!
- Boundary value problems – Involves solving solutions of large numbers of equations; *efficiency* is key concern.

ATMS 502 - Spring 2019 A011: PDE type/classification; A012: PDE characteristics 2/28/19

Characteristics

30

- Hyperbolic case; *characteristic curves*
  - Example: 2-D wave equation
  - Two wave speeds: ±*c*
  - An initial value problem



Adapted from Figure 2-6 of Anderson et al., p. 23

- Anderson et al. textbook: *"A fundamental property of hyperbolic PDEs is the limited domain of dependence" [shown in figure at left]*
- Analytical solution at (x_0,t_0) depends *only* on data between the characteristic curves including *initial data* between (x_0-ct_0), (x_0+ct_0)

ATMS 502 - Spring 2019 A012: PDE characteristics; A014: Domain of dependence 2/28/19

CFL

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- The CFL (Courant-Freidrichs-Lewy) condition
  - "On the partial difference equations of mathematical physics" *
  - Based on the *data* that
    - determines the solution to the actual PDE
    - determines the solution in the numerical scheme
  - Requires that
    - the numerical domain of dependence must *include the* PDE domain of dependence.
- A *necessary but not sufficient* condition for stability.
  - The CFL criteria need not agree with the results of the Von Neumann analysis.

*Courant, R., K. O. Fredrichs, and H. Lewy (1928), "Uber die Differenzengleichungen der Mathematischen Physik", Math. Ann, vol.100, p.32, 1928.

ATMS 502 - Spring 2019 A014: PDE Domain of dependence • C038: CFL condition • C039: Numerical domain of dependence 2/28/19

Domain of Dependence

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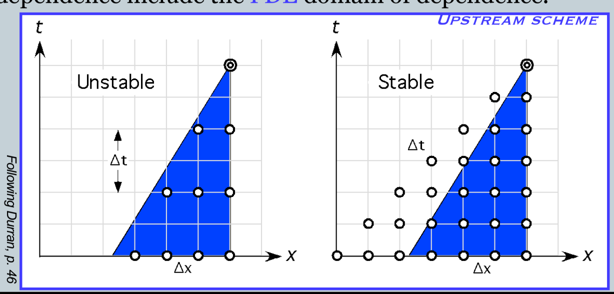
- The CFL condition requires that the numerical domain of dependence include the PDE domain of dependence.



Following Durran, p. 46

Unstable case: true domain of dependence extends outside of numerical one.
Stable case: Δt *halved*; numerical domain contains true domain of dependence.

ATMS 502 - Spring 2019 A014: PDE Domain of dependence • C038: CFL condition • C039: Numerical domain of dependence 2/28/19